SELECTED PROBLEMS AND SOLUTIONS (2017-2021)

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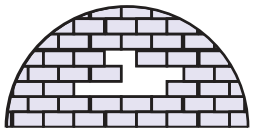
Pre-Ecolier

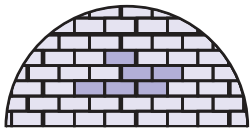
PRE-ECOLIER 2017 Q6

How many bricks like a this ▭ are missing in the igloo?



(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

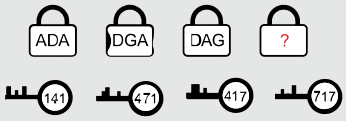
Answer. (A)
By inserting the bricks into the empty parts, we get



Therefore, 1 + 2 + 3 = 6 bricks are missing in the igloo.

PRE-ECOLIER 2017 Q18

Each of the 4 keys fits one of the 4 padlocks. The numbers on the keys refer to the letters on the padlocks



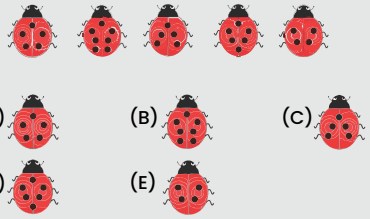
What is written on the last padlock?

(A) GDA (B) ADG (C) GAD (D) GAG (E) DAD

Answer. (D)
From the first three padlocks and keys, we know that the letters *A*, *D* and *G* correspond to the digits 1, 4 and 7 respectively, and vice versa. Hence three–digit number 717 matches to the letter GAG.

PRE-ECOLIER 2018 Q6

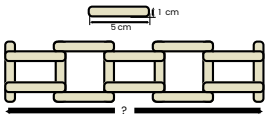
Which of these ladybirds has to fly away so that the rest of them have 20 dots in total?



Answer. (B)
By adding the number of dots, these ladybirds have a total of 5 + 7 + 5 + 6 + 4 = 27 dots on the back.
To attain a total of 20 dots, we have to remove a ladybird who had 27 - 20 = 7 dots on the back.

PRE-ECOLIER 2018 Q18

A student had some sticks of length 5 cm and width 1 cm. Using the sticks he constructed the fence below.
What is the length of the fence?



(A) 20 (B) 21 (C) 22 (D) 23 (E) 25

Answer. (B)
Observe that the fence has a length which is equivalent to the 5 lengths with 4 overlapping width of the sticks, that gives 5 × 5 - 4 × 1 = 21 cm.

PRE-ECOLIER 2019 Q5

At the entrance of the zoo there are 12 children in the queue. Lucy is the seventh from the front and Kim is the second from the back. How many children are there between Lucy and Kim in the queue?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Answer. (B)
Based on the description, there are 7 - 1 = 6 and 12 - 2 = 10 children standing in front of Lucy and Kim, respectively.
Then there are a total of 10 - 7 = 3 children standing between them.

Pre-Ecolier

PRE-ECOLIER 2019 Q20

3 squirrels Anni, Asia and Elli collected 7 nuts in total. Each collected a different quantity of nuts, but each collected at least one. Anni collected the least and Asia collected the most. How many nuts did Elli collect?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

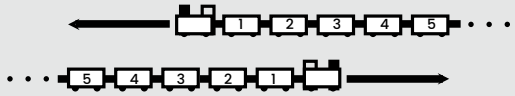
Answer. B
We consider the partition of 7 nuts into three parts:

$$7 = 1 + 1 + 5 = 1 + 2 + 4 = 1 + 3 + 3 = 2 + 2 + 3$$

Since they own distinct number of nuts, the only possible assignment is giving 1, 2 and 4 nuts to Anni, Elli and Asia, respectively.

PRE-ECOLIER 2020 Q15

Two identical trains, each with 31 cars, are traveling in opposite directions. When car No. 19 of one train is opposite car No. 19 of the other, which car is opposite car No. 12?

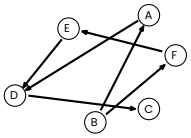


(A) 7 (B) 12 (C) 21 (D) 26 (E) 31

Answer. D
The sum of numbers on the car of two identical opposite trains is always constant. Hence, the opposite of the car No. 12 is 19 + 19 - 12 = 26.

PRE-ECOLIER 2020 Q17

An arrow pointing from one person to another means that the first person is taller than the second. For example, person B is taller than person A.



Who is the shortest?

(A) Person A (B) Person B (C) Person C (D) Person D (E) Person E

Answer. C
We see that the height in descending order are B, A, D, C and B, F, E, D, C, in which two paths showing that person C is less dominance among other five persons, so he is the shortest among others.

PRE-ECOLIER 2021 Q11

Julia has two pots with flowers, as shown. She keeps the flowers exactly where they are. She buys more flowers and puts them in the pots. After that each pot has the same number of each type of flower.

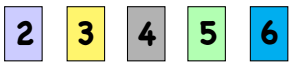
(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

What is the smallest number of flowers she needs to buy?

Answer. C
To attain the minimum number of flowers to be brought, Julia needs to buy 4 - 2 = 2 white flowers, 4 - 1 = 3 gray flowers and 3 - 2 = 1 black flower. Therefore, she needs to buy 2 + 3 + 1 = 6 flowers in total.

PRE-ECOLIER 2021 Q19

The cards shown in the picture are placed into two boxes. The sums of the numbers in each box are the same. Which number must be in the box with the number 4?



(A) Only 3 (B) Only 5 (C) Only 6
(D) Only 5 or 6 (E) Impossible to determine

Answer. C
The total sum of these numbers is 2 + 3 + 4 + 5 + 6 = 20, so each box has cards that sum up to 20 ÷ 2 = 10. This implies that the other card in the box with number 4 must have a total value of 10 - 4 = 6, and this is attainable when you put the card numbered 6 in the box.

Ecolier

ECOLIER 2017 Q9

Balloons are sold in packets of 5, 10 and 25. Marius buys exactly 70 balloons. What is the smallest number of packets he could buy?

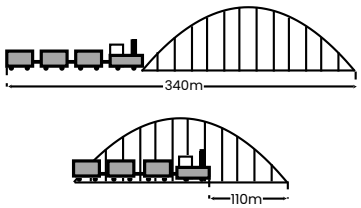
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Answer. (B)

To obtain the minimum number of packets to be brought, Marius must buy the packet of 25 as many as possible, followed by the packets of 10 and 5. At first, he had bought. $\frac{70}{25}=2$ packets of 25 balloons, then followed by $\frac{70-2\times 25}{10}=2$ packets of 10 balloons and 0 packet of 5 balloons. This guarantees Marius could buy the minimum number of packets, that is 2 + 2 + 0 = 4 packets.

ECOLIER 2017 Q16

How long is the train?



(A) 55 (B) 115 (C) 170 (D) 220 (E) 230

Answer. (B)

The figure above shows that the train has traveled 340-110 = 230 m, from the initial position to the mid-way of the bridge, which is equivalent to the length of two trains. Hence, the train is 230 ÷ 2 = 115 m long.

ECOLIER 2018 Q7

Diana takes part in an archery contest. On her first turn, Diana scores 12 points with three arrows (left figure). On her second turn, she scores 15 points (middle figure). How many points does she score on her third turn (right figure)?

(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

Answer. (D)

An increment of 15 - 12 = 3 points is obtained when Diana moves her arrow from outer to inner region. If she shoot all 3 arrows to the inner region, his final score will be increased by 3×3 = 9 points compared to her initial score (first turn). Therefore, she scored 12+ 3×3 = 21 points on her third turn.

ECOLIER 2018 Q20

In a video game, the final enemy is an evil robot with many heads. To defeat the robot, we have to cut all its heads. If we cut 3 heads, one new head will grow. We can only defeat the robot by cutting off 13 heads in total. How many heads did the robot have in the beginning?

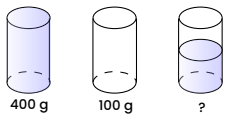
(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

Answer. (B)

Since $3^2 + 3^1 + 3^0 = 13$, so this implies that the robot initially has $3^2 = 9$ heads.

ECOLIER 2019 Q15

A full glass of water weighs 400 grams. An empty glass weighs 100 grams. How many grams does a half-full glass of water weigh?



(A) 100 (B) 200 (C) 225 (D) 250 (E) 300

Answer. (D)

A full glass of water without glass weighs 400 - 100 = 300 grams, so half-full glass of water without glass weighs 300 ÷ 2 = 150 grams. It follows that a half-full glass of water weighs 150 + 100 = 250 grams.

Ecolier

ECOLIER 2019 Q19

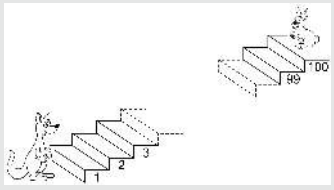
The pages of a book are numbered 1, 2, 3, 4, 5, and so on. The digit 5 appears exactly 16 times. What is the maximum number of pages this book could have?

(A) 49 (B) 64 (C) 66 (D) 74 (E) 80

Answer. (B)
We listed out the page numbers that contain the digit 5: 5, 15, 25, 35, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, ⋯ If the digit 5 appears exactly 16 times in this book, then the book may have 59, 60, 61, 62, 63 or 64 pages by counting the appearance of digit 5 in the list above. The maximum number of pages is thus 64.

ECOLIER 2020 Q13

Every time the kangaroo goes up 7 steps, the rabbit goes down 3 steps.



(A)53 (B) 60 (C) 63 (D) 70 (E) 73

Answer. (D)
They have made a total of 7 - (-3) = 10 steps when they climbed the stairs once together. This implies that they have to climbed the stairs for 100÷10 = 10 times before they meet each other. It follows that the kangaroo has jumped to $7 \times 10 = 70^{th}$ stair.

ECOLIER 2020 Q19

Several teams came to the summer Kangaroo camp. Each team has 5 or 6 members. There are 43 people in total. How many teams are at this camp?

(A) 4 (B) 6 (C) 7 (D) 8 (E) 9

Answer. (D)
Note that we can have at most $\left[\frac{43}{5}\right] = 9$ teams of 5 and $\left[\frac{43}{6}\right] = 8$ teams of 6. We listed down some possible number of teams in the table below.

#Team of 5	9	8	7	6	5	4	3	⋯
#Team of 6	0	0	1	2	3	4	5	⋯
Total number of people	45	40	41	42	43	44	45	⋯

The column in blue shows that only 5 teams of 5 and 3 teams of 6, with a total of 5 + 3 = 8 teams participating the camp.

ECOLIER 2021 Q20

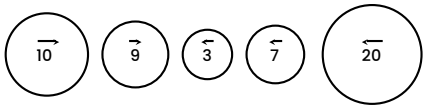
A box has fewer than 50 cookies in. The cookies can be divided evenly between 2, 3, or 4 children. However, they cannot be divided evenly between 7 children because 6 more cookies would be needed. How many cookies are there in the box?

(A) 12 (B) 24 (C) 30 (D) 36 (E) 48

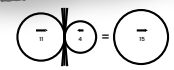
Answer. (D)
The number of cookies must be a multiple of 2, 3 and 4, so is the multiple of gcd (2, 3, 4) = 12. This leaves the possibility of the number of cookies to be 12, 24, 36 and 48. However, a cookie would be remained when distribute to 7 children, so the box has only 36 cookies by eliminating the above three possibilities.

ECOLIER 2021 Q12

The 5 balls shown begin to move simultaneously in the directions indicated by their arrows.



When two balls going in opposite directions collide, the bigger ball swallows the smaller one and increases its value by the value of the smaller ball. The bigger ball continues to move in its original direction, as shown in the following example.



What is the final result of the collisions of the 5 balls shown?

(A) 50 (B) 48 (C) 49 (D) 50 (E) 49

Answer. (C)
First, the balls numbered 9 and 3 collide each other and give a ball scores 9 + 3 = 12 towards the right, then it collides with another ball whose score is 7 and moving towards the left so that they become a ball with score 12 + 7 = 19 towards the right. It follows that the ball numbered 19 towards the right will be swallowed by the biggest ball, resulted in a new ball with score 20 + 19 = 39 travel to the left direction. Last, it collides with the ball whose score is 10 and finally the ball is moving towards left and having a score of 49.

Benjamin

BENJAMIN 2017 Q7

A special cube has a number on each face. The sums of the numbers on opposite faces are all equal. Five of the numbers are 5, 6, 9, 11 and 14. What number is on the sixth face?

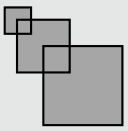
(A) 4 (B) 7 (C) 8 (D) 13 (E) 15

Answer. E

The sum of all numbers in all faces must be a multiple of 3 as sums of the numbers on opposite faces are all equal. Since 5 + 6 + 9 + 11 + 14 = 45 is a multiple of 3, this forces the last number to be a multiple of 3 too. We can count each of the choices modulo 3 and find out only 15 gives the correct answer. Alternatively, we need to find two pairs that have the same sum. By trying, the two pairs are 6 + 14 and 9 + 11. Since both sums are equal to 20, the number on the opposite side of 5 must be 20 - 5 = 15.

BENJAMIN 2017 Q15

Rafael draws three squares. The first one has side length 2 cm. The second one has side length 4 cm and a vertex is placed at the centre of the first square. The last one has side length 6 cm and a vertex is placed at the centre of the second square. What is the area of the figure?



(A) 32 cm^2 (B) 51 cm^2 (C) 27 cm^2 (D) 16 cm^2 (E) 6 cm^2

Answer. B

The first square has an area of 2 × 2 = 4 cm2, the second square has an area of 4 × 4 = 16 cm2, and third square has an area of 6 × 6 = 36 cm2. The overlapping of the first square and second square is one-fourth of the area of the first square, and the overlapping of the second square and third square is one-fourth of the area of the second square. Therefore, the total area is 4 + 16 + 36 - $\frac{1}{4}\cdot(4+16)=51cm^2$

BENJAMIN 2018 Q12

Two girls, Eva and Olga and three boys, Adam, Isaac and Urban play a ball game. When a boy has the ball, he throws it to another boy but never to the boy from whom he just received it. Eva starts the game by throwing the ball to Adam. Who will do the fifth throw?

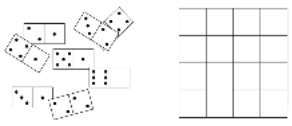
(A) Adam (B) Eva (C) Issac (D) Olga (E) Urban

Answer. A

Following the rules, Adam, who received the ball from Eva, should pass over the ball to either Issac or Urban for the third and fourth turn as boys only pass the ball among themselves. This forces Adam to get the ball in the fifth throw.

BENJAMIN 2018 Q24

There are eight domino tiles on the table. Half of one tile is covered. The 8 tiles can be arranged into a 4 × 4 square, so that the number of dots in each row and each column is the same. How many dots are on the covered part?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Answer. C

By adding the number of dots along each column or row, the total number of dots must be a multiple of 4. Since 0 × 1 + 1 × 4 + 2 × 4 + 3 × 2 + 4 × 2 + 5 × 1 + 6 × 1 = 37 ≡ 1 (mod 4), so the covered part must has some number of dots congruence to 3 modulo 4. Then the possible number of dots among the choices is 3.

BENJAMIN 2019 Q24

The sum of the dots on opposite faces of a regular dice is equal to 7. Which of the following shows the regular dice?

(A)  (B)  (C)  (D)  (E) 

Answer. E

Notice that the sum of opposite faces to be 7 can be done through pairing 1 with 6, 2 with 5 and 3 with 4. Apparently, we can only see the dice with dots numbered either (1, 2, 3), (1, 2, 4), (1, 5, 3), (1, 5, 4), (6, 2, 3), (6, 2, 4), (6, 5, 3) or (6, 5, 4) in the view for regular dice, and only dice (E) satisfies the condition above.

Benjamin

BENJAMIN 2019 Q24

Zev has two machines: one exchanges 1 white token into 4 red tokens, while the other exchanges 1 red token into 3 white token. Zev has 4 white tokens. After 11 exchanges, he has 31 tokens. How many of those are red?

(A) 21 (B) 17 (C) 14 (D) 27 (E) 11

Answer. C
Since the number of tokens obtained in final is invariant within specific steps, denote Zev has made x white token into red tokens and y red token into white tokens throughout the process. It follows that 4+3x+2y = 31 since increments of 3 and 2 tokens occurred when Zev exchanges from white to red and red to white tokens respectively. Moreover, we have x + y = 11 as 11 exchanges are occurred. Solving for x and y gives x = 5 and y = 6. This concludes that there are 5 × 4 − 6 = 14 red tokens. The following is one of the examples to get 31 tokens in 11 exchanges:

$$4R \xrightarrow{3\text{ exchanges}} 1W+12R \xrightarrow{6\text{ exchanges}} 19W+6R \xrightarrow{2\text{ exchanges}} 17W+14R$$

BENJAMIN 2020 Q6

Kim has several chains of length 5 and of length 7.

By joining chains one after the other, Kim can create different lengths. Which of these lengths is impossible to make?

(A) 10 (B) 12 (C) 13 (D) 14 (E) 15

Answer. C
We check each choice carefully: 10 = 5 × 2, 12 = 5 + 7, 14 = 7 × 2 and 15 = 5 × 3. 13 is the only number here that could not be partitioned into parts of 5 and 7.

BENJAMIN 2021 Q10

There is a square with line segments drawn inside it. The line segments are drawn either from the vertices or the midpoints of other line segments. We colored $\frac{1}{8}$ of the large square. Which one is our coloring?

Answer. D
By calculating the area of each shaded region, we have.

$$S_{\text{red}} = \left(\frac{1}{8}\right)^2 = \frac{1}{64}\ \text{unit}^2$$

$$S_{\text{blue}} = \frac{1}{2}\cdot\frac{1}{8}\cdot\frac{1}{4} = \frac{1}{64}\ \text{unit}^2$$

$$S_{\text{yellow}} = \frac{1}{2}\cdot 1\cdot\frac{1}{2} = \frac{1}{4}\ \text{unit}^2$$

$$S_{\text{green}} = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 = \frac{1}{8}\ \text{unit}^2$$

$$S_{\text{orange}} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}\ \text{unit}^2$$

Therefore, only shaded region in yellow gives an area of $\frac{1}{8}$ unit^2.

BENJAMIN 2021 Q18

There were 20 apples and 20 pears in a box. Carl randomly took 20 pieces of fruit from the box and Luca took the rest. Which of the following statements is always true?

(A) Carl got at least one pear
(B) Carl got as many apples as pears
(C) Carl got as many apples as Luca
(D) Carl got as many pears as Luca got apples
(E) Carl got as many pears as Luca

Answer. D
Since the number of Carl's and Luca's apples equals 20 and the number of Carl's apples and Luca's pears also equals 20. Hence, Carl got as many pears as Luca got apples. Other options is not always true.